Shaped Sinewaves

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Abstract

A sine wave may be skewed or distorted in various ways. One possible application in audio is for an LFO to add some flexibility to vibrato, tremolo, etc.

1 Introduction

Sometimes a simple Sine-LFO does not allow convincing musical expression such as vibrato, tremolo, etc. Maybe the rising and falling slopes need to be diferent, or likewise the dwell times at the upper and lower turning points, respectively. Sometimes even a combination of both skewness and asymmetry is desirable in adjustable mounts. This note offers simple mappings with parameters to provide such flexibility.

2 Skewness

The first goal is to allow a sine waveform to be skewed. We may achieve this by combining the sine with its associated cosine in the following way.

Observe that

$$\frac{\sin\phi}{\sqrt{1+\cos\phi}} = s\left|\sin\tfrac{\phi}{2}\right| \tag{1}$$

and

$$\frac{\sin\phi}{\sqrt{1-\cos\phi}} = s\left|\cos\tfrac{\phi}{2}\right|, \tag{2}$$

where $s = \sqrt{2}\,\mathrm{sign}\,(\sin\phi)$. One may say that in equation (1) the sine is skewed to the right, while in eq.(2) it is skewed to the left. Now introduce a parameter q between -1 and $+1$, then the following expression will gradually morph between the two extremes:

$$\frac{\sin\phi}{\sqrt{1+q\cos\phi}} \tag{3}$$

However, we want the result to be bounded between -1 and $+1$, therefore we need to normalize. A short calculation fixes it:

$$u(\sin\phi, \cos\phi) = \sin\phi\sqrt{\frac{A}{1+q\cos\phi}}, \quad \text{where } A = \frac{1+\sqrt{1-q^2}}{2}. \tag{4}$$

We may write the result in a slightly different way. With the substitution $q = 2t/(1+t^2)$ we obtain

$$u(s, c) = \frac{s}{\sqrt{1+2tc+t^2}}, \tag{5}$$

where we have used abbreviations s and c for $\sin\phi$ and $\cos\phi$, respectively.

Figure 1 shows two cycles of waveform u for selected values of t.

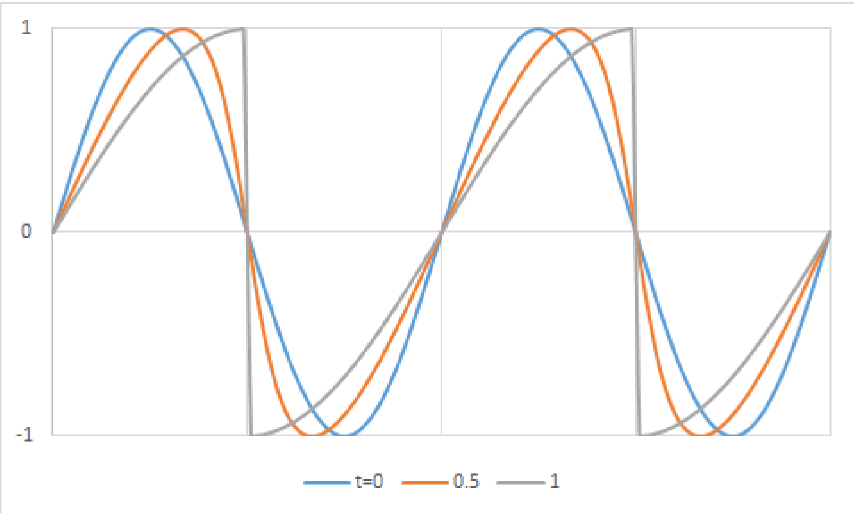

Figure 1: Skewed sinewave, eq.(5), for selected values of t.

3 Asymmetry

Observe that

$$\sqrt{1+\cos\phi} = \sqrt{2}\,\left|\cos\tfrac{\phi}{2}\right| \tag{6}$$

and

$$\sqrt{1-\cos\phi} = \sqrt{2}\,\left|\sin\tfrac{\phi}{2}\right| \tag{7}$$

The idea is that the expression

$$\sqrt{1+p\cos\phi} \tag{8}$$

with parameter p between -1 and $+1$ should morph nicely between the two above extremes. To make the result be bounded by ±1 we need to stretch and shift the expression

$$v = A\big(\sqrt{1+p\cos\phi} - B\big) \quad \text{with } A = \frac{\sqrt{1+p}+\sqrt{1-p}}{p} \text{ and } B = \frac{\sqrt{1+p}+\sqrt{1-p}}{2}. \tag{9}$$

However, this representation is ill-defined for $p = 0$. We need to regularize! Remapping $p = 2r/(1 + r^2)$ and some algebra yields the following equivalent expression

$$v(c) = \frac{2c + r}{1 + \sqrt{1 + 2rc + r^2}}. \tag{10}$$

Figure 2 shows two cycles of waveform v for selected values of r.

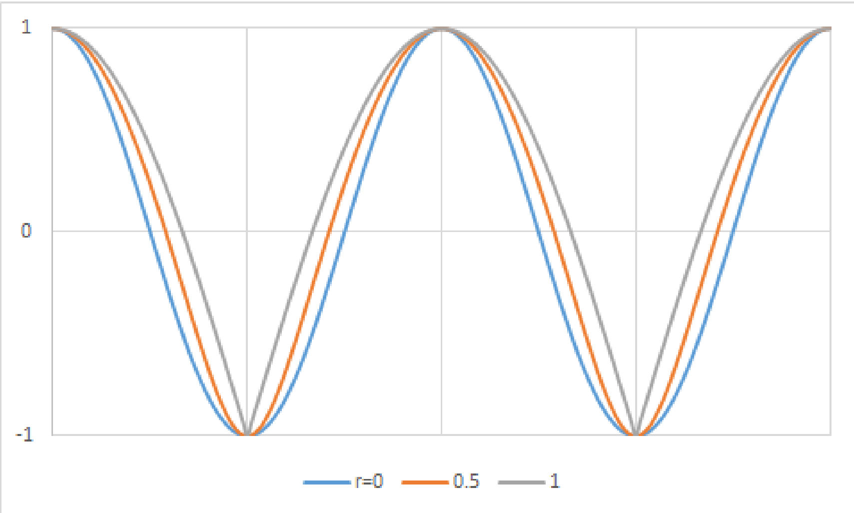

Figure 2: Asymmetric waveform, eq.(10), for selected values of r.

4 Combined skewness and asymmetry

Since the function v maps the interval $[-1, +1]$ onto itself, we might just as well feed it with a sine instead of cosine. In fact, we might chain the transformations effected by u and v to obtain

$$w(s, c) = v\big(u(s, c)\big). \tag{11}$$

The mapping function w incorporates both parameters t and r for skewness and asymmetry, respectively. Figures 3 and 4 show some results.

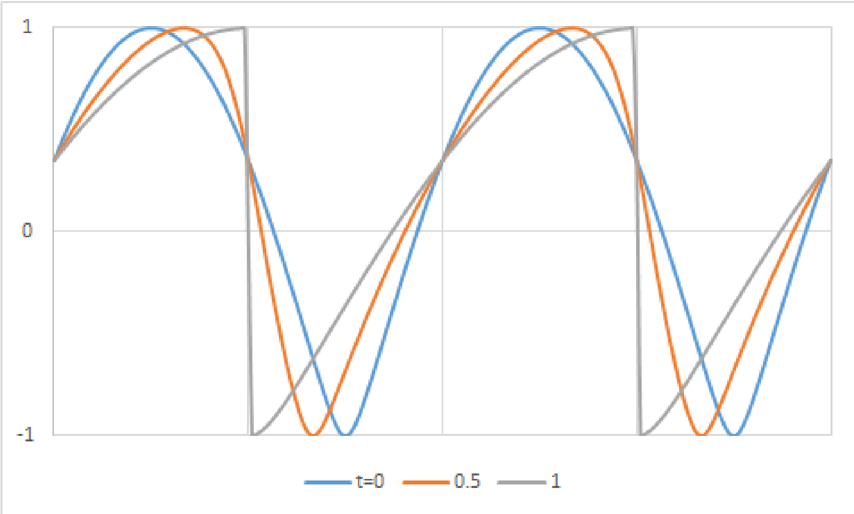

Figure 3: Skewed asymmetric waveform, eq.(11), for selected values of the skewness parameter t. The asymmetry parameter is kept constant at $r = 0.8$.

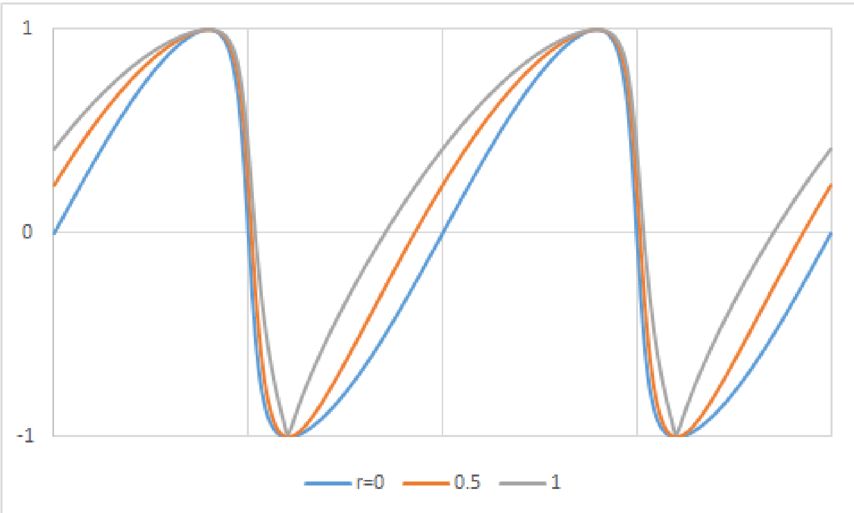

Figure 4: Skewed asymmetric waveform, eq.(11), for selected values of the asymmetry parameter r. The skewness parameter is kept constant at $t = 0.8$.